Universal extra dimensions

Kağan Şimşek

Department of Physics
Middle East Technical University
Ankara, Turkey
ksimsek@metu.edu.tr

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General motivation: Grand unification schemes

- String theories
- 10-, 11-, or 26-dimensions

Personal motivation: Top physics

Top quark:

- Main decay mode:

$$t \to bW \quad (BR \simeq 1)$$

- Rare decay modes:

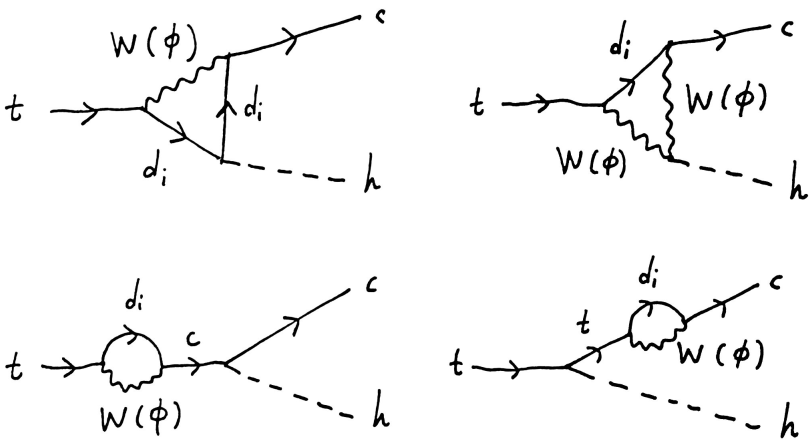
$$t \to ch \quad (BR = 2 \times 10^{-14})$$

$$t \to c\gamma \quad (BR = 2 \times 10^{-13})$$

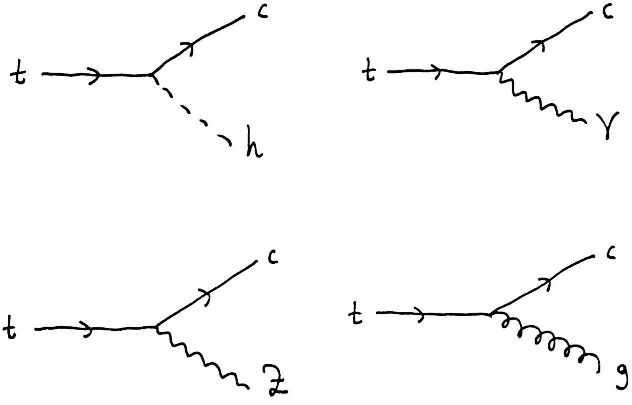
$$t \to cZ \quad (BR = 7 \times 10^{-14})$$

$$t \to cg \quad (BR = 2 \times 10^{-11})$$

$t \to ch$:



Interactions absent at tree level:



Process	Theoretical BR	Experimental BR
$t \to ch$	2×10^{-14}	$< 2.2 \times 10^{-3}$ (95%)
$t \to c\gamma$	2×10^{-13}	$< 1.7 \times 10^{-3}$ (95%)
$t \to cZ$	7×10^{-14}	$< 0.49 \times 10^{-3}$ (95%)
$t \to cg$	2×10^{-11}	

Can we account for these large orders of gap in an extra-dimensional scenario?

Standard Model Lagrangian:

$$\mathscr{L} = \mathscr{L}_{\text{gauge}} + \mathscr{L}_{\text{gauge fixing}} + \mathscr{L}_{\text{ghosts}} + \mathscr{L}_{\text{higgs}} + \mathscr{L}_{\text{fermion}} + \mathscr{L}_{\text{yukawa}}$$

where

$$\mathscr{L}_{\text{gauge}} = \sum_{V=g^a,W^i,B} -\frac{1}{4}V^{\mu\nu}V_{\mu\nu}$$

: kinetic terms for mediators

$$\mathscr{L}_{\text{gauge fixing}} = \sum_{a=1}^{8}\left[-\frac{1}{2}(\partial_\mu g^{a\mu})^2\right] + \sum_{i=1}^{3}\left[-\frac{1}{2}(\partial_\mu W^{i\mu} - m_W\phi^i)^2\right] + \left[-\frac{1}{2}(\partial_\mu B^\mu - m_B\phi^3)^2\right]$$

: terms fixing redundant degrees of freedom

$$\mathscr{L}_{\text{higgs}} = |\mathscr{D}_\mu H|^2 + \mu^2|H|^2 - \lambda|H|^4$$

: terms giving mediators their masses

$$\mathscr{L}_{\text{fermion}} = \sum_{f=Q,U,D,L,E} \bar{f} i \gamma^{\mu} \mathscr{D}_{\mu} f$$

: kinetic terms of fermions

$$\mathscr{L}_{\text{yukawa}} = -y_e \bar{L} E H - y_u \bar{Q} U \tilde{H} - y_d \bar{Q} D H + \text{h.c.}$$

: terms giving fermions their masses

Conventions:

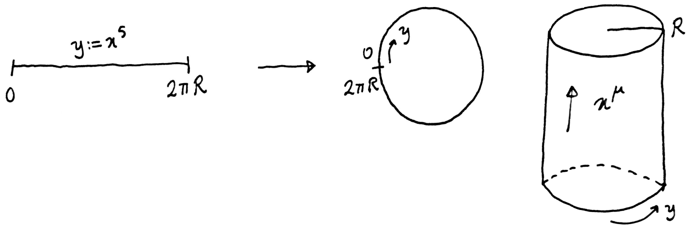
$$g^{\mu\nu} = + - --$$

$$\mathscr{D}^{\mu} = \partial^{\mu} + \frac{ig_s}{2}\vec{\lambda}\cdot\vec{g}^{\mu} + \frac{ig_w}{2}\vec{\tau}\cdot\vec{W}^{\mu} + \frac{ig_y}{2}YB^{\mu}$$

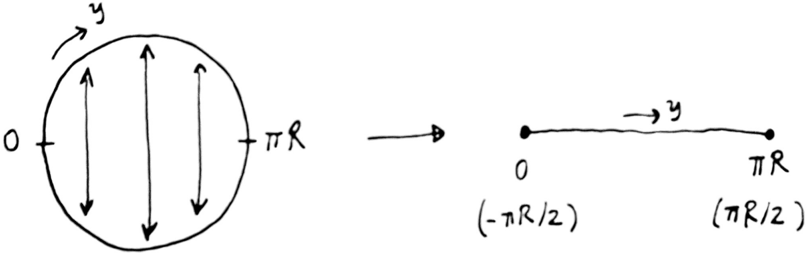
$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} i(\phi^1 + i\phi^2) \\ h + v + i\phi^3 \end{pmatrix}$$

Promotion to $5D$:

$$\mu, \nu, \ldots = 0, 1, 2, 3 \quad \rightarrow \quad M, N, \ldots = 0, 1, 2, 3, 5$$

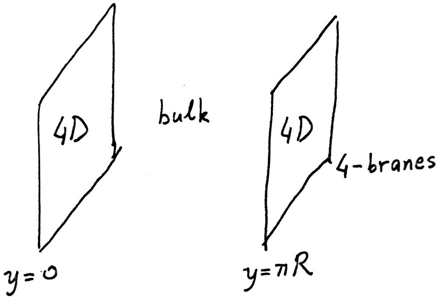


S^1/Z_2 orbifold:



The orbifolding is required to obtain chiral fermions in $5D$.

Minimal vs. nonminimal UED:



The nonminimal model contains boundary localized terms at the action level:

$$\mathscr{L} \supset [\delta(y) + \delta(y - \pi R)] \left[\frac{1}{2} r \left(\partial_\mu \phi \right)^2 - \frac{1}{2} m_b^2 \phi^2 \right]$$

Back to minimal UED:

Z_2 symmetry $\Rightarrow$ new conserved quantum number, KK parity

The case of a massive scalar:

$$S = \int d^5x \left[\frac{1}{2}(\partial_M \phi)^2 - \frac{1}{2} m_5^2 \phi^2 \right]$$

m_5 : $5D$ mass term, a combo of $5D$ couplings and Higgs VEV

Equation of motion:

$$(\pentagon + m_5^2)\phi(x,y) = 0$$

$$\pentagon := \partial_M \partial^M = \Box - \partial_y^2$$

Ansätz:

$$\phi(x,y) = \sum_{n \geq 0} \phi_n(x) f_n(y) : \text{ KK tower of } 5D \text{ field } \phi$$

$$(\Box + m_n^2)\phi_n(x) = 0, \quad f_n''(y) + M_n^2 f_n(y) = 0, \quad m_n^2 := M_n^2 + m_5^2$$

Solution:

$$\phi(x,y) = \sum_{n \geq 0} \phi_n(x)(A_n \sin M_n y + B_n \cos M_n y)$$

Z_2 symmetry:

$$\phi_\pm(x,y) = \pm\phi_\pm(x,-y)$$

Therefore,

$$\phi_+(x,y) = \sum_{n \geq 0} \phi_{+n}(x) \cos M_n y$$

$$\phi_-(x,y) = \sum_{n \geq 0} \phi_{-n}(x) \sin M_n y$$

Boundary conditions:

$$\phi\Big|_{y=0,\pi R} = 0 \quad \text{or} \quad \partial_y \phi\Big|_{y=0,\pi R} = 0$$

If the particle has a counterpart in SM, then impose Neumann condition.
If it is new to SM, then impose Dirichlet condition.

$$\phi_-\Big|_{y=0,\pi R} = 0 \quad \text{and} \quad \partial_y \phi_+\Big|_{y=0,\pi R} = 0$$

This give the same mass quantization for both types of fields:

$$\phi_+(x,y) = \sum_{n\geq 0} \phi_{+n}(x) \cos \frac{ny}{R}$$
$$\phi_-(x,y) = \sum_{n\geq 1} \phi_{-n}(x) \sin \frac{ny}{R}$$

Repeat the procedure for all the scalars, fermions, and gauge bosons.

Apparently, we have infinitely many particles (KK partners). By vacuum stability analysis, the maximum KK number to use is

$$n_{\max} = 6$$

KK expansion of all the fields, normalized on the interval $y \in [0, \pi R]$:

$$\phi_+(x,y) = \frac{1}{\sqrt{\pi R}}\phi_{+0}(x) + \sum_{n\geq 1} \phi_{+n}(x)\sqrt{\frac{2}{\pi R}}\cos\frac{ny}{R}$$

$$\phi_-(x,y) = \sum_{n\geq 1} \phi_{-n}(x)\sqrt{\frac{2}{\pi R}}\sin\frac{ny}{R}$$

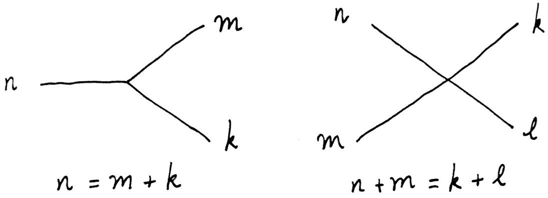
$$V^\mu(x,y) = \frac{1}{\sqrt{\pi R}}V^\mu_-(x) + \sum_{n\geq 1} V^\mu_n(x)\sqrt{\frac{2}{\pi R}}\cos\frac{ny}{R}$$

$$V^5(x,y) = \sum_{n=1} V^5_n(x)\sqrt{\frac{2}{\pi R}}\sin\frac{ny}{R}$$

$$f_L(x,y) = \frac{1}{\sqrt{\pi R}}f_{L0}(x) + \sum_{n\geq 1} \sqrt{\frac{2}{\pi R}}\left[P_L f_{Ln}(x)\cos\frac{ny}{R} + P_R f_{Ln}(x)\sin\frac{ny}{R}\right]$$

$$f_R(x,y) = \frac{1}{\sqrt{\pi R}}f_{R0}(x) + \sum_{n\geq 1} \sqrt{\frac{2}{\pi R}}\left[P_R f_{Rn}(x)\cos\frac{ny}{R} + P_L f_{Rn}(x)\sin\frac{ny}{R}\right]$$

Put all the fields into the $5D$ action and integrate over the extra dimension. This gives selection rules for possible vertices:



This is related to the fifth component of the momentum being conserved due to translational symmetry.

There is an accidental symmetry. Let

$$T : y \to y + \pi R$$

Then

$$\phi_n \to \lambda_n \phi_n$$

where $\lambda_n = (-1)^n$ is called the KK parity. If

$$(-1)^{n \pm m \pm k} = 1 \quad \text{for a 3-point vertex}$$

$$(-1)^{n \pm m \pm k \pm \ell} = 1 \quad \text{for a 4-point vertex}$$

the Lagrangian is invariant under T, hence the KK parity is conserved.

Caution! Couplings are dimensionful:

$$\int_0^{\pi R} dy\ \bar{f} i \gamma^\mu \mathscr{D}_\mu f \supset -\frac{g_5}{\sqrt{\pi R}} \bar{f}_0 \gamma^\mu f_0 V_\mu \supset -g \bar{f}_0 \gamma^\mu f_0 V_\mu$$

Thus, $[g_5] = M^{-1/2}$. Therefore, the theory is nonrenormalizable, and we need a cutoff, Λ.

$$\Lambda R = n_{\max} = 6$$

and we obtain an effective $4D$ Lagrangian.

Spontaneous symmetry breaking in the effective Lagrangian:

Scalars

Gauge basis	Mass basis
g_n^5	g_n^5
$W_n^{15}, W_n^{25}, \phi_n^1, \phi_n^2$	$G_n^\pm, a_n^\pm$
$W_n^{35}, B_n^5, \phi_n^3$	G_n^1, G_n^2, a_n

Fermions

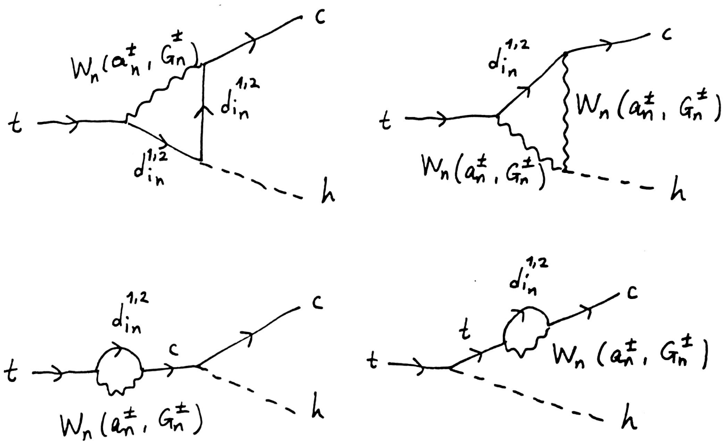
Gauge basis	Mass basis
f_{Ln}, f_{Rn}	f_{1n}, f_{2n}

Vectors

Gauge basis	Mass basis
g_n^μ	g_n^μ
$W_n^{1\mu}, W_n^{2\mu}$	$W_n^{\pm\mu}$
$W_n^{3\mu}, B_n^\mu$	A_n^μ, Z_n^μ

The SM spectrum remains the same in the minimal UED.

New diagrams for $t \to ch$:



For all the fields,

$$m_n = \sqrt{m^2 + \frac{n^2}{R^2}}$$

Thus, the spectrum is highly degenerate.
The radiative corrections[1], which induce boundary localizations, breaks down the degeneracy in the spectrum.

[1] They are ignored in this work.

Coding:

- LanHEP model by Belyaev *et al.* (2015)
- Dubious vertex factors
- Original Mathematica code to check vertex factors

(* FFS/nn0 and FFV/nn0 interactions of quarks in our MUED *)

```
Z1 = 0; (* set this 1 to get all Z's 1. -- this turned out to be superfluous. *)

(* γ^5 and P_R/L are always to the right of γ^μ. *)
```

(* spectrum *)

(*

up: SM mode of up–like Dirac quark;
down: SM mode of down–like Dirac quark;
up1: KK mode of 1st up–like Dirac quark;
up2: KK mode of 2nd up–like Dirac quark;
down1: KK mode of 1st down–like Dirac quark;
down2: KK mode of 2nd down–like Dirac quark;

Wp: SM mode of W^+ boson;
Wm: SM mode of W^- boson;
Z: SM mode of Z boson;
A: SM mode of photon;
g: SM mode of gluon ;

Wpn: KK mode of W^+ boson;
Wmn: KK mode of W^- boson;
Pn: KK mode of P boson;
Vn: KK mode of V boson;
gn: KK mode of gluon ;

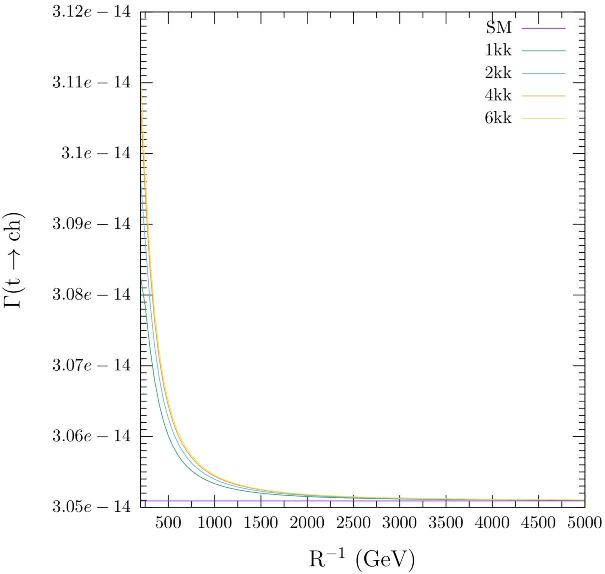
higgs: SM mode of Higgs ;
Wpf: SM mode of Goldstone of W^+ boson ;
Wmf: SM mode of Goldstone of W^- boson;
Zf: SM mode of Goldstone of Z boson ;

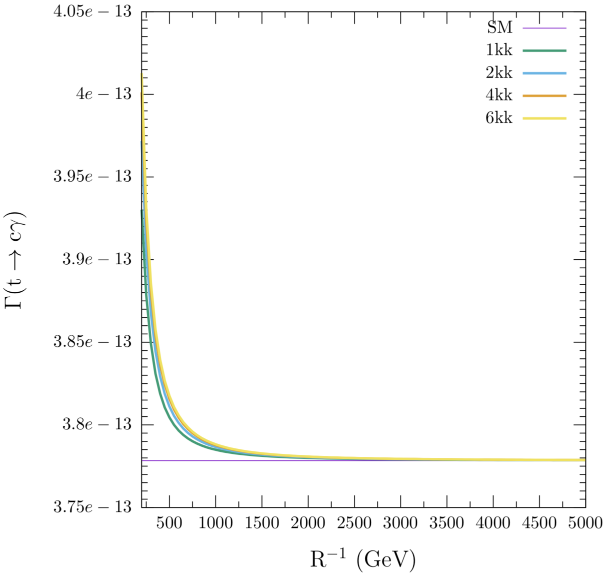
Wpnf: KK mode of Goldstone of W^+ boson ;
Wmnf: KK mode of Goldstone of W^- boson;
Pnf: KK mode of Goldstone of P_n boson;
Vnf: KK mode of Goldstone of V_n boson;
gnf: KK mode of Goldstone of gluon ;

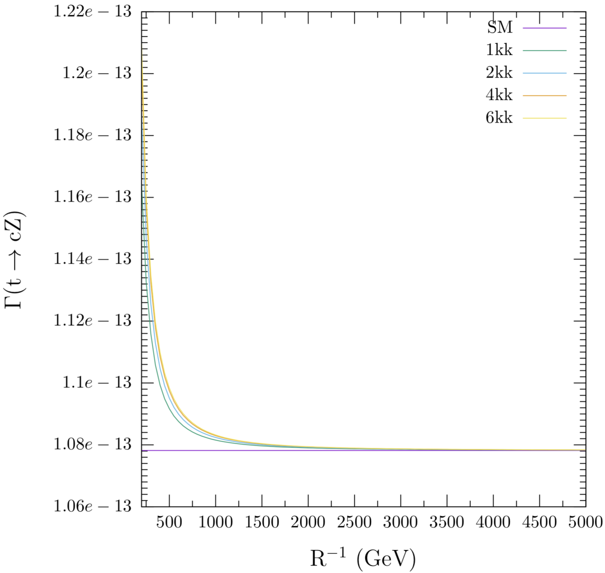
hn:KK mode of Higgs;
apn and amn: charged scalars in the tower;
an: neutral scalar in the tower;

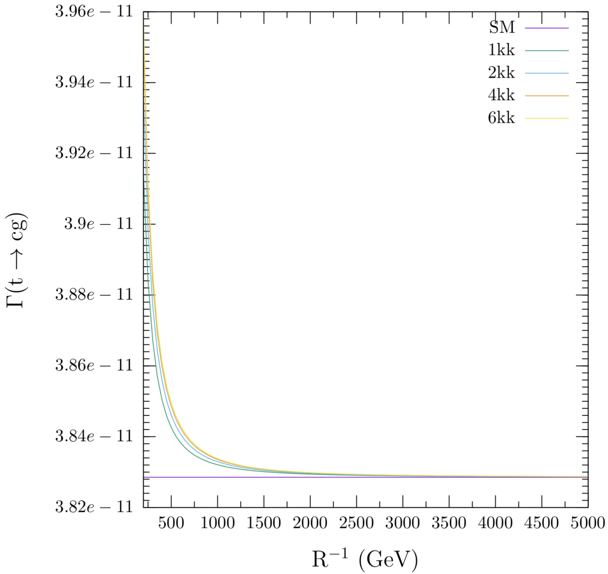
*)

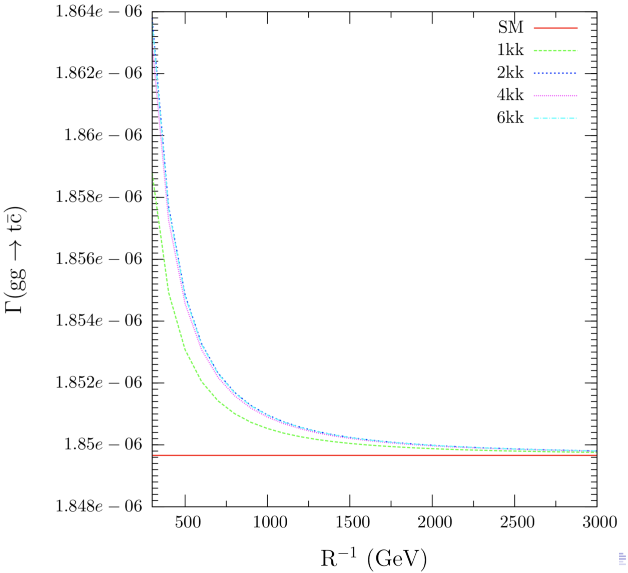
```
(* usage *)

vertex[MainField_, FermionBar_, Fermion_] :=
 If[MemberQ[{Wp, Wm, Z, A, g, Wpn, Wmn, Pn, Vn, gn}, MainField], vec[MainField, FermionBar, Fermion],
    sca[MainField, FermionBar, Fermion]] /. rule2 // Simplify
```


3.12e − 14
3.11e − 14
3.1e − 14
3.09e − 14
3.08e − 14
3.07e − 14
3.06e − 14
3.05e − 14
Γ(t → ch)
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
R−1 (GeV)
SM
1kk
2kk
4kk
6kk


4.05e − 13
4e − 13
3.95e − 13
3.9e − 13
3.85e − 13
3.8e − 13
3.75e − 13
Γ(t → cγ)
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
R^{−1} (GeV)
SM
1kk
2kk
4kk
6kk


1.22e − 13
1.2e − 13
1.18e − 13
1.16e − 13
1.14e − 13
1.12e − 13
1.1e − 13
1.08e − 13
1.06e − 13
Γ(t → cZ)
SM
1kk
2kk
4kk
6kk
500 1000 1500 2000 2500 3000 3500 4000 4500 5000
R⁻¹ (GeV)


3.96e − 11
3.94e − 11
3.92e − 11
3.9e − 11
3.88e − 11
3.86e − 11
3.84e − 11
3.82e − 11
Γ(t → cg)
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
R^−1 (GeV)
SM
1kk
2kk
4kk
6kk


SM
1kk
2kk
4kk
6kk
1.864e − 06
1.862e − 06
1.86e − 06
1.858e − 06
1.856e − 06
1.854e − 06
1.852e − 06
1.85e − 06
1.848e − 06
500
1000
1500
2000
2500
3000
Γ(gg → tc̄)
R−1 (GeV)

- Contributions are very small.
- Model is predictable – only free parameter is R.
- Dark matter candidates: g_1^{μ} and A_1^{μ} are the lightest KK particles whose stability is guaranteed by KK parity.

What is next: Nonminimal UED

- Boundary localized terms

$$\mathscr{L} \supset [\delta(y) + \delta(y - \pi R)] \left[\frac{1}{2} r \left(\partial_\mu \phi \right)^2 - \frac{1}{2} m_b^2 \phi^2 \right]$$

- Loss of KK number conservation
- Extended parameter space: r and m_b for each field
- Richer phenomenology

Original paper:

- T. Appelquist, H.-C. Cheng, & B. A. Dobrescu, “Bounds on Universal Extra Dimensions” [arXiv:hep-ph/0012100]

Minimal UED coding:

- A. Belyaev, M. Brown, J. Moreno, & C. Papineau, “Discovering Minimal Universal Extra Dimensions (MUED) at the LHC” [arXiv:1212.4858]

Electroweak symmetry breaking and gauge-fixing in nonminimal UED:

- T. Flacke, A. Menon, & D. J. Phalen, “Non-minimal universal extra dimensions” [arXiv:0811.1598]
- T. Flacke, K. Kong, & S. C. Park, “A Review on Non-minimal Universal Extra Dimensions” [arXiv:1408.4024]

Similar literature studies:

- U. J. Dey, & T. Jha, “Rare Top Decays in Minimal and Non-minimal Universal Extra Dimension” [arXiv:1602.03286]
- C.-W. Chiang, U. K. Dey, & T. Jha, “$t \to cg$ and $t \to cZ$ in Universal Extra Dimensional Models” [arXiv:1807.01481]