Mechanics Labs with iOLab

for Phys 113/121

Kağan Şimşek

Summer 2020

UNIVERSITY of ROCHESTER

Acknowledgment

The author acknowledges the amount of work and effort accomplished before him to the best of his knowledge.

The original contents, published as iOLab for Scientists and Engineers, have been developed by Tom Hemmick.

The template of the lab report has been inspired by the one that Prof. Frank Wolfs uses in the course Phys 141.

v. 202006171248CT

Contents

Introduction

Inside the box

When you have obtained the iOLab device, you should find the following in the box:

- One device/remote and one dongle (Figure 1), and
- One screwdriver, one long spring, one eyebolt, two felt pads, one plate, and one short spring (Figure 2).

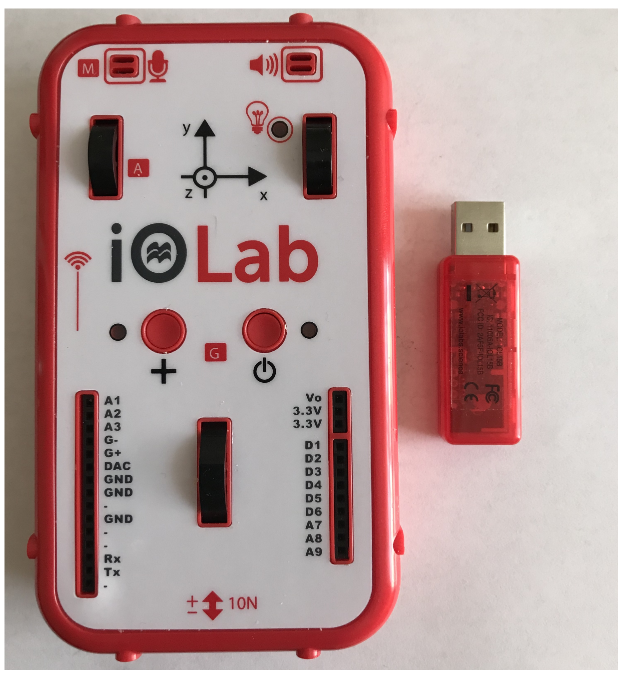


Figure 1: The primary contents of the iOLab device. From left to right, the device/remote and the Dongle.

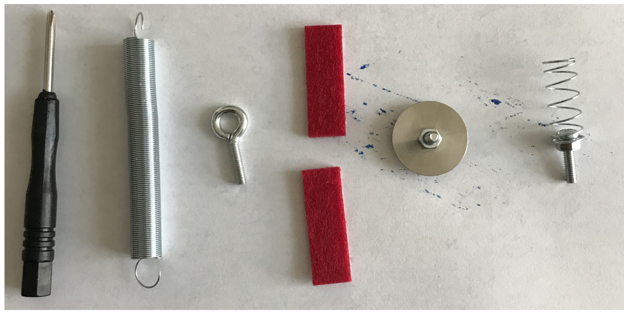


Figure 2: The secondary contents of the iOLab device. From left to right, the screwdriver, the long spring, the eyebolt, the felt pads (two of them), the plate, and the short spring.

Driver and iOLab application

Please visit www.iolab.science to download the driver (Windows only) and the iOLab application (Windows and Mac). Follow the instructions for the installation.

Pairing

When you have downloaded the iOLab application, launch it. Insert the Dongle into the computer. While pressing on and holding the + button, power on the device. Hold until the bulb next to the + button lights up in triplets (three fast dots). Select `Remote 1` in the application, and it should automatically pair the remote.

Calibration

You must calibrate the device on every computer you use in correspondence with the iOLab device. In order to calibrate the device, make sure your device is turned on and the dongle is plugged into your computer. Click on the tools button in top right corner of the page (small gear icon). This will give you a dropdown menu.

One of the options in this menu is `Calibration`. You will then be able to click on either `Accel - magn - gyro` or `Force` under `Calibration`.

You will do two calibrations: one for the `Force` sensor and one for the `Accelerometer - Magnetometer - Gyroscope`. You will need to attach the eyebolt to the force probe in order to complete the calibration of the force probe.

Once you select a calibration, simply follow the directions given. The calibrations are a pretty quick process, but it is necessary to do in order to ensure you are getting accurate data. Once you are done with the calibration, make sure you hit the save button.

Taking data

Open the iOLab software. On the left hand side of the page, you will see a menu with all the possible sensors (with check boxes next to each). To take data, first make sure the dongle is plugged into the computer and the device is turned on. Then check the boxes of the sensors you would like to use. There are limitations, but you can use multiple sensors at the same time.

In this example, we will be using the sensors `Force` and `Accelerometer`. Watch this video to find out how to get started. You can try it out for yourself on the right hand side of this screen. The sensors have already been selected for you but you can control when you start and stop the recording, the smoothing, the y axis, and the zooming function. By selecting the `Remove` button above all the plots, this will clear the plots and allow you to retake the data if you would like to.

Analyzing the data

After you take the data, there is some analysis to do. Watch this video to see an example of analysis in action. The following are the instructions in words.

Looking at the software page where you took the data, there are three connected buttons that look like a bar graph, a magnifier, and two arrows orthogonal to each other. The "bar graph" is the analysis mode. The middle one is the zoom button. It has actually a dropdown menu: The first option allows you to choose a box section of the plot on which to zoom. The second option allows you to zoom horizontally only, and the last option allows you to zoom vertically only. To unzoom, simply click on the plot while in Zoom mode. The last button in this block of three allows you to move the position of the plot.

The analysis mode of the software allows you to highlight specific sections of the plots to analyze. When a section of the plot is highlighted, the following information will be given to you: μ, σ, a, and s:

- μ is the average value of the highlighted portion.
- σ is the uncertainty in the highlighted portion.
- a is the area under the curve.

- s is the slope of the highlighted section.

Saving data

All data that you take will be saved within the software. If you take data in the normal iOLab application, just click on the folder button and you will see all the data you have taken in the order you have taken it. The most recent data will be at the top of the list and each item in the list will be time stamped and marked by which sensors were used. However, keep in mind that any highlighting you do under analysis mode will not be saved.

If you take data within a lab manual, you will be able to access it by selecting the "Show reports" link underneath each lab manual title. Once you click on that link, your old reports will show up in a box. By clicking on the box of the report you want, you can open your lab manual with the data that you have taken. In addition, if you take data within the lab manual and you want to re-take the data, you can hit the remove button on top of the plot in order to erase the data and start fresh.

Error analysis

There are three major types of error that we deal with in science:

- Instrument precision
- Unknown error
- Known error

For the purposes of this lab, we will only be dealing with the first two types.

Instrument precision

Every time you use a measuring apparatus such as a ruler, there is a level of uncertainty associated with reading the value. Two people might not read the same exact value and you might not read the same value if you conducted the measurement multiple times. For errors like this, there is usually a known precision of that instrument. This is usually half the smallest divison of the instrument, or the smallest increment in which you can accurately read. For example, the smallest division on a ruler is 1 mm. Half of that is 0.5 mm. Therefore, the error in a measurement from a ruler would be $\pm$0.5 mm.

Unknown error

There are other errors that do not stem from the instrument itself, but in the way the experiment is conducted. In these cases, the error is not necessarily known, but it is possible to estimate them. To do that, you have to repeat the same experiment in the same conditions multiple times. You should find that the measured values are spread about a mean value, μ. The spread from the mean, σ, is called the standard deviation. This will be the range in which the "true value" is likely to lie, with the most probable value being the mean. Here are the formulae associated with the mean and the standard deviation:

$$\mu = \sum_i \frac{x_i}{N}$$
$$\sigma^2 = \sum_i \frac{(x_i - \mu)^2}{N-1}$$

where N is the number of trials and x_i is the value measured during that trial.

Note that for future labs, Δ will be used to denote the error in a quantity. For example, the error in x is Δx.

Compatibility

This manual is 100% self-contained that you can download (and even printout, and then you can use a QR code reader to access the links) and view all the original YouTube videos. The procedure is explained carefully step by step for each experiment, completing possible missing steps in the original one. You should be able to complete each experiment by following the steps written here and write a proper lab report.

If you really want, then you can always read the original manual that comes with the device. There is only one problem with the original manual: It does not tell you what to record or how to write your lab report. Eventually, you need to go over this manual, again.

Writing a lab report

There are certain data (graphs and tables) you will record and analyze by using the iOLab application. However, there is no lab report format offered with iOLab. Therefore, I want you to prepare a lab report by using the template in Figure 3.

In the title, please do not keep the "#" symbol. To illustrate, in your first lab report, if you are taking Phys 113, the title should look like

Experiment 1: Force and Acceleration Kagan Simsek Phys 113 June 17, 2020

You may use a word processor, a spreadsheet application, a Mathematica notebook, LaTeX, or any other means to type it. Or, you may simply write it by hand (*and it is your responsibility for your handwriting to be legible*), scan the pages, and create a pdf.

Please, if you want to write the report by hand, do not submit a zip file containing lots of photos generically named like `IMG_2881.jpg`, `IMG_2882.jpg`, etc. It is very important for you to create a pdf out of your scanned image files.

For the submission of your lab report, I have prepared submission links at the end of each experiment (See here, here, here, here, and here). The lab reports that are not submitted in this way are highly likely to get lost in my inbox, hence they will not be graded.

Experiment #: Title of the experiment
Student name
Phys 113 or Phys 121
Date

Abstract
Summarize your main results here. Be brief and concise.

I. THEORY
Include the motivation and objectives of the experiment. Describe the theory behind it. Mention the key formulae, explaining the meaning of the symbols used. One paragraph should be sufficient.

II. EXPERIMENT DETAILS
Briefly describe the apparatus you are using. Write down the procedure in your own words – do not copy from the manual coming with the iOLab. Describe the steps as if you are explaining to someone who will do it for the first time. If necessary, draw small figures, but you don't have be artistic.

III. DATA ANALYSIS AND RESULTS
Perform the **TASKS** here by using subheadings as in

Task 1 Do exactly what Task 1 tells you to do.

Task 2 Do exactly what Task 2 tells you to do.

While completing tasks, explain all the plots and the steps in your calculations. Sometimes, you will need to prepare tables. You can prepare them by using a spreadsheet application and copy it to your report. Do not forget the units.

IV. CONCLUSION AND DISCUSSION
Quote your final result(s) here and put a box around them or prepare a small table and put the final result(s) there. If you obtain more than one results for one variable (e.g. mass, the coefficient of friction, to name a few), it is this place to compare them. Discuss if your results agree with the theory.

Sometimes there will be tasks to complete specifically in the discussion section. Do them here, using task subheadings again.

Figure 3: The format of the lab report.

Experiment 1

Force and acceleration

1.1 Objectives

In this experiment, we are going to

- study the relation between force and acceleration, and
- use the Newton 2$^{\text{nd}}$ Law to find the mass of your iOLab device.

1.2 Useful links

Here are some links that contain useful information about the theory behind this experiment and instructions on how to take and analyze data.

- Block on a ramp
- Force 12
- Pushes on device
- Finding the mass of the device
- Parametric plot

1.3 Force and acceleration: Qualitative comparison

Make sure the iOLab application is up and running and your remote is paired. Insert the eyebolt into the force probe. Select the sensors `Accelerometer` and `Force`. Start recording data, and give the remote four pushes along the y axis. Stop recording data.

TASKS

(1) What do you notice about the positions of the peaks in the F vs. t plot and the peaks in the a vs. t plot? Include a screenshot of your data with the zoom selected to make all data for both force and acceleration easy to read. Make a qualitative statement about whether F and a occur at the same time and whether they grow smaller/larger in a correlated manner.

1.4 Finding the "known" value of the mass

The mass of the iOLab device will be measured in later again, hence you should learn how to perform this section quickly. Please, in the next experiment, do not use the data from this experiment.

Reset any previous runs. Insert the eyebolt into the force probe. Put the remote on the table so that the y axis is downwards and start recording data. Wait two seconds. Now grab the remote by the eyebolt and lift it. Hold steady for two seconds again. Put it back. Stop recording data.

TASKS

(2) Take a screenshot of the plots. Include them in your report.

(3) By using the analysis tool (to the left of the magnifier), select the "valley" in the force-time graph and record the average force and its uncertainty. Include the screenshot of your plot with correct highlights.

(4) By using the analysis tool, select the acceleration in the force-free region and record the average acceleration and its uncertainty. Include the screenshot of your plot with correct highlights.

(5) Compute the mass of the remote.

1.5 Quantitative measurement of F and a

1.5.1 Part I

Reset any previous runs. Replace the eyebolt in the force probe with the plate. Put the device on the table so that the wheels point toward the ceiling. Start recording data. Give the device five consecutive shoves of increasing strength. Stop recording data.

TASKS

(6) Deselect Ax and Az in the acceleration-time plot, zoom into the plots such that you see all the five peaks. Include the screenshot of your plots.

(7) By using your cursor, find the peak values of a and F for each shove. Record these values in a neat table, clearly indicating the units.

(8) Plot your data (F vs. a with acceleration in the horizontal and force in the vertical) by using your favorite tool and add a linear trendline.

(9) Find the slope of the trendline, and hence find the mass. Compare this mass value to the "known" value.

1.5.2 Part II

Reset any previous runs. Remove the plate in the force probe. Attach the eyebolt to the force probe, and attach the long spring to it. Let the device hang from the spring vertically securely. When it is nearly in equilibrium, start recording data. Give a small pull and let it oscillate vertically for a while. Stop recording data.

TASKS

(10) Make a parametric plot of a vs. F. Make sure that force is in the vertical.

There is also one essential point to remark here. The acceleration here is the norm, namely

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} \tag{1.1}$$

but clearly we don't want this. Click on the small button at the bottom left corner of the graph (the one with a small arrow on its top right corner) and set the norm to Ay.

TASKS

(11) In order to measure the slop, select the analysis tool and highlight the sine-like waves at the bottom of the graph. Include the screenshot of your plot with correct highlights.

However, there is no tool within the iOLab application to measure this slope. You need to switch to the zoom mode, hover (don't click) on five points on this line to compute the slope.

TASKS

(12) Get five data points on the line, tabulate your data, and compute the slope.

(13) Determine whether the masses are in agreement.

1.6 Error analysis

To compute the "known" value of the mass, you used average values of force and acceleration, $F_{\rm av}$ and $a_{\rm av}$. These necessarily contain errors, ΔF and Δa. Therefore, the mass also contains some error. By using the rule of division, you can compute it very easily:

$$\Delta m = \frac{F_{\rm av}}{a_{\rm av}}\left(\frac{\Delta F}{F_{\rm av}} + \frac{\Delta a}{a_{\rm av}}\right) \tag{1.2}$$

By using this uncertainty, discuss if the value of the mass you found in Parts I and II lies between the error limits.

TASKS

(14) Compute Δm and discuss if the value of the mass you found in Parts I and II lies between the error limits. Find the percent difference between the value from Part I and the known value, as well as the value from Part II and the known value.

1.7 Discussion

In the Conclusion and Discussion section of your lab report, include answers to the following among other things (see Figure 3):

TASKS

(15) How do you find the mass from the slope of the plots?

(16) Do the values for mass you found in Parts I and II match the known value?

1.8 Submitting lab report

In order not to experience any difficulties with the submission, please name your file as

```
p121_exp1_lastname_firstname.pdf
```

or

```
p113_exp1_lastname_firstname.pdf
```

minding the underscores. Then, click here to submit your paper [1].

[1] If the link does not work, please send your pdf to `kagannsimsek@gmail.com` by typing the subject of the email exactly as Mechanics Experiment 1. You may leave the body of the email empty.

Experiment 2

Force of friction

2.1 Objectives

In this experiment, we are going to

- study how the force of friction changes with mass, and
- solve for the coefficient of kinetic friction between the device and the surface.

2.2 Useful links

Here are some links that contain useful information about the theory behind this experiment and instructions on how to take and analyze data.

- Tied blocks
- Static friction
- Finding the mass of the device

2.3 Physics overview

Friction force opposes the sliding of an object. It comes in two types: static and kinetic. The static one prevents motion until a strong enough external force acts on the object to set it in motion. The kinetic one comes into play once the object start moving. You may have observed in daily life that you need a greater force to set an object in motion than to keep it moving, hence the static friction cannot be less than the kinetic one.

In this experiment, we will be putting additional mass on the remote and pushing it around on a level surface. The normal force is

$$N = mg \tag{2.1}$$

and the friction force (of either type) is given by

$$F_f = \mu N \tag{2.2}$$

Then, it is possible to find μ here. Note that it is most likely for each student to come up with a different μ value, for it depends only on the materials which make up the device and the surface.

2.4 Finding the "known" value of the mass

The mass of the iOLab device will be measured in later again, hence you should learn how to perform this section quickly. Please, in the next experiment, do not use the data from this experiment.

Reset any previous runs. Insert the eyebolt into the force probe. Put the remote on the table so that the y axis is downwards and start recording data. Wait two seconds. Now grab the remote by the eyebolt and lift it. Hold steady for two seconds again. Put it back. Stop recording data.

TASKS

(1) Take a screenshot of the plots. Include them in your report.

(2) By using the analysis tool (to the left of the magnifier), select the "valley" in the force-time graph and record the average force and its uncertainty. Include the screenshot of your plot with correct highlights.

(3) By using the analysis tool, select the acceleration in the force-free region and record the average acceleration and its uncertainty. Include the screenshot of your plot with correct highlights.

(4) Compute the mass of the remote, and the uncertainty in it.

2.5 Give the device a push

Reset any previous runs. Remove the eyebolt from the force probe and insert the plate. We will again need the sensors `Accelerometer` and `Force`. Put the device on the table with the wheels up. Start recording data.

TASKS

(5) Give the device a push in the y direction. Stop recording data. Uncheck `Ax` and `Az` boxes. Increase smoothing if you want, but that is not necessary. Include the screenshot of your plot.

(6) When you stop pushing the device, it still has an acceleration. Why? Using the mass and the acceleration, how can you find the force acting on the device?

Let's call this part of the graph the "from friction" region.

2.6 Procedure

Prepare three small objects to tape onto the device. We will need them one by one. Note that you should tape the objects on the front side of the remote that has wheels.

Let's start with the first object. Tape it onto the remote.

TASKS

(7) Find the mass of this new system.

(8) Using the sensors `Accelerometer` and `Force`, give the system a push and obtain the acceleration-time and force-time plots. Include them in the report.)

(9) By using the analysis tool, highlight the acceleration in the region where only the friction force is in effect. Record the average acceleration and the uncertainty in it in this region. Include the screenshot of the plot with correct highlights.

(10) Use the force of gravity to compute the normal force.

(11) Compute the force of friction from the acceleration.

(12) Compute the coefficient of friction, μ.

Take the second object. Tape it on top of the previous system.

TASKS

(13) Find the mass of this new system.

(14) Using the sensors `Accelerometer` and `Force`, give the system a push and obtain the acceleration-time and force-time plots. Include them in the report.)

(15) By using the analysis tool, highlight the acceleration in the region where only the friction force is in effect. Record the average acceleration and the uncertainty in

it in this region. Include the screenshot of the plot with correct highlights.

(16) Use the force of gravity to compute the normal force.

(17) Compute the force of friction from the acceleration.

(18) Compute the coefficient of friction, μ.

Finally, take the third object. Tape it on top of the previous system.

TASKS

(19) Find the mass of this new system.

(20) Using the sensors `Accelerometer` and `Force`, give the system a push and obtain the acceleration-time and force-time plots. Include them in the report.)

(21) By using the analysis tool, highlight the acceleration in the region where only the friction force is in effect. Record the average acceleration and the uncertainty in it in this region. Include the screenshot of the plot with correct highlights.

(22) Use the force of gravity to compute the normal force.

(23) Compute the force of friction from the acceleration.

(24) Compute the coefficient of friction, μ.

By now, you should have

- Three different values for the normal force,
- Three different values for the friction force, and
- Three different values for the coefficient of friction.

TASKS

(25) Prepare a neat table with the columns N, F_f, and μ. Don't forget the units. Plot the force of friction vs. the normal force with the former in the vertical and the latter in the horizontal, and add a linear trendline.

(26) Calculate the average and standard deviation of the tabulated values of μ.

(27) Using the slope of the graph you have just plotted, obtain μ.

(28) Compare the two values of μ.

2.7 Error analysis

You should be able to tell that the only force in the "from friction" region is the force of friction. This is why you were able to compute the friction force in a straightforward manner above. However, a word of notice is essential here. All the mass an acceleration terms contain errors. (If you didn't record the errors above, then please go back and repeat the necessary parts.) By using the rule of multiplication, the force of friction will have an error

$$\Delta F_f = ma_f \left(\frac{\Delta m}{m} + \frac{\Delta a_f}{a_f} \right) \tag{2.3}$$

By using these errors, you can estimate the uncertainty in the magnitude of the friction force.

While you are trying to obtain the gravitational force, F_g, indeed you are measuring the normal force – in magnitude! In those tasks, you should have noted the uncertainties in the gravitational acceleration so that you can compute the error in the gravitational forces as

$$\Delta F_g = ma \left(\frac{\Delta m}{m} + \frac{\Delta a}{a} \right) \tag{2.4}$$

where a is indeed the gravitational acceleration here. Now that you have the uncertainties for each value of F_f and N (via F_g), you can insert the error bars in your plot.

TASKS

(29) Insert the error bars in the plot of F_f vs. N. Include the graph in your report.

2.8 Discussion

In the Conclusion and Discussion section of your lab report, include the following among other things (see Figure 3):

TASKS

(30) Discuss your results if they agree with expectations.

(31) Draw a free-body diagram of forces acting on the device for all four times: (i) Just after you give the remote a push, (ii) just after the remote gains motion, (iii) just after you release your force agent (most possibly your finger), and (iv) just after the remote stops.

(32) Clearly explain what the slope of the plot of F_f vs. N represents.

2.9 Submitting lab report

In order not to experience any difficulties with the submission, please name your file as

`p121_exp2_lastname_firstname.pdf`

or

`p113_exp2_lastname_firstname.pdf`

minding the underscores. Then, click here to submit your paper [1].

[1] If the link does not work, please send your pdf to `kagannsimsek@gmail.com` by typing the subject of the email exactly as Mechanics Experiment 2. You may leave the body of the email empty.

Experiment 3

Circular motion

3.1 Objectives

In this experiment, we are going to

- measure the angular velocity of the device when spinning in the air,
- measure the tangential velocity of the point labeled A (just to the right of the upper left wheel on the iOLab device), and
- use the relations between the linear and rotational motion to find the radius of rotation of the point labeled A and compare this to the actual value.

3.2 Useful links

Here are some links that contain useful information about the theory behind this experiment and instructions on how to take and analyze data.

- Gravitron
- Chair swing
- Martini swirl
- Circular motion video 1
- Circular motion video 2
- Circular motion video 3

- Circular motion video 4

3.3 Physics overview

Just as x denotes the linear distance traveled by a moving body, θ measures the rotation in units of radian.

All the points on a uniformly spinning object – say, a sphere rotating about the axis that passes through its poles – have the same angular velocity, ω. However, the tangential speed of each point will be different, at least for the points at a different radial distance to the center of rotation, which is apparent in the equation

$$v = \omega r \tag{3.1}$$

In circular motion, the magnitude of the tangential velocity remains the same but its direction changes all the time. Since the velocity vector changes, there is an acceleration, called the *centripetal acceleration*, a_c, and it is directed toward the center of the circle – perpendicular to the tangential velocity. The size of the centripetal acceleration is given by

$$a_c = \frac{v^2}{r} \tag{3.2}$$

where r is the radius of the circular motion and v is the tangential velocity.

In our setup, the *fulcrum*, namely the center of rotation, is located at the center of mass of the iOLab device. This center of mass is at the point `G` in the front face of the device at the center. During the experiment, we will be throwing the device up into the air so that it rotates about the z axis. During this motion, the accelerometer of the remote – labeled by the point `A` – will measure the acceleration as it sweeps out a circle around the center of mass. Using the sensors `Gyroscope` and `Accelerometer` in the iOLab application, the angular velocity and linear acceleration at this point will be found. With this information, can find the distance between the points `A` and `G`.

3.4 How are we doing this?

Connect the remote and open the application. Select the sensors `Gyroscope` and `Accelerometer` in the column on the left-hand side. Take the remote in your hands. You may want to practice a bit just to ensure that you can catch the remote safe and sount (accidents may happen, be careful).

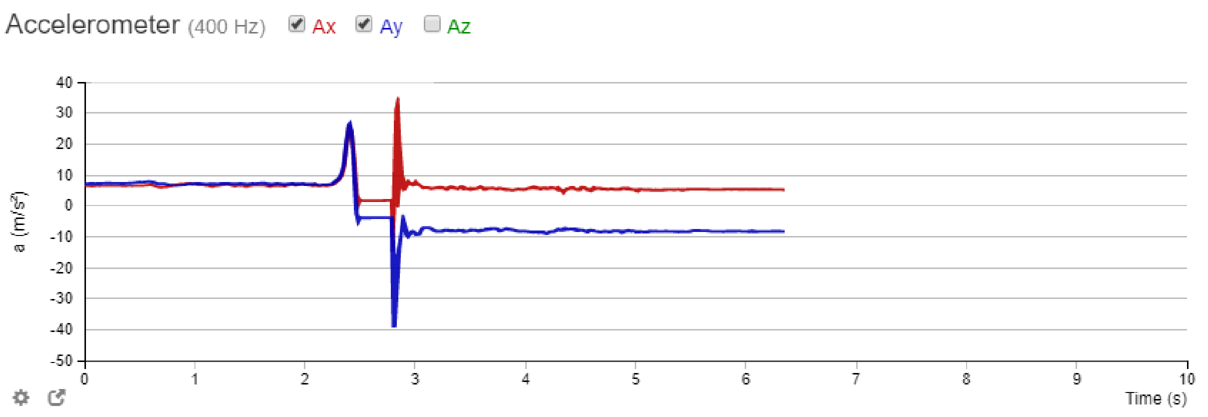
Now, we will throw the remote in the air. Doing so, we need to be careful as to the following:

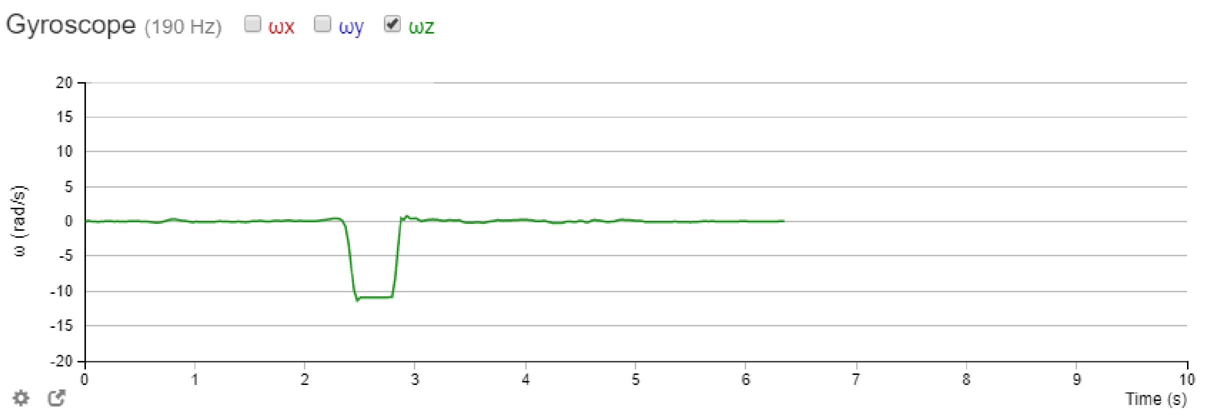
- You should hold the remote so that the z direction either points to the left or to the right in all trials – be persistent in your choice.
- When you throw the remote in the air, try to rotate it as much as possible about the z axis and as little as possible about the other axes.
- In the first throw, give it some slight angular velocity. In the second, make it rotate faster while you throw it. The third one should be the fastest. If the angular velocities do not come out ordered, just exchange data between the runs.
- We will do three different runs, so record one data, analyze it, and when you are finished, go to the next one.
- You don't need to throw it 50 cm above or up to the ceiling.. Just ensure that it rotates a bit in the air before you catch it.

When you are ready, start recording and throw the remote in the air by giving it a little rotation.

3.5 Analysis: Vector addition

An example run is shown below.





Since the linear motion takes place in the x and y directions, we only keep those components of the acceleration. Since the rotation takes place about the z axis, we only keep the z component of the angular velocity. Adjust the limits of the vertical axes and see the full range.

It is apparent that only during the time interval for which the angular velocity forms a "valley" do we have the rotation. The rest of the data is before and after the rotation. Let's crop them out by using the zoom tool. Then, highlight these regions and read the average and standard devication values for a_x, a_y, and ω_z.

We need the magnitude of the linear acceleration:

$$a = \sqrt{a_x^2 + a_y^2} \tag{3.3}$$

Compute this value by using the average values.

After this little preview, let's start.

3.6 Procedure

On the front face of the remote, you see two points labeled `A` and `G`. We have the accelerometer sensor at `A` and the gyroscope at `G`. The point `G` also indicates the center of mass of the remote.

TASKS

(1) Measure the distance between the points `A` and `G`.

You will throw the remote in the air with increasing angular velocities. Let's start with the smallest one. Hold the remote on your hands as described previously. Start recording data.

TASKS

(2) Throw the remote in the air, catch it, stop recording data, and obtain the graphs by the accelerometer and the gyroscope. Select the relevant components, namely `Ax` and `Ay` in the acceleration-time plot and ω`z` in the angular velocity. Zoom into the part of the graph during which the remote is spinning in the air – the "valley" in the gyroscope graph. Select the analysis tool and record the average values and the errors for a_x, a_y, and ω_z. Include the screenshot of the plot with correct highlights in the analysis mode.

(3) Compute the magnitude of the total centripetal acceleration, a_c.

Now, repeat for the second run.

TASKS

(4) Throw the remote in the air, catch it, stop recording data, and obtain the graphs by the accelerometer and the gyroscope. Select the relevant components, namely `Ax` and `Ay` in the acceleration-time plot and ω`z` in the angular velocity. Zoom into the part of the graph during which the remote is spinning in the air – the "valley" in the gyroscope graph. Select the analysis tool and record the average values and the errors for a_x, a_y, and ω_z. Include the screenshot of the plot with correct highlights in the analysis mode.

(5) Compute the magnitude of the total centripetal acceleration, a_c.

Now, repeat for the third run.

TASKS

(6) Throw the remote in the air, catch it, stop recording data, and obtain the graphs by the accelerometer and the gyroscope. Select the relevant components, namely `Ax` and `Ay` in the acceleration-time plot and ω`z` in the angular velocity.

Zoom into the part of the graph during which the remote is spinning in the air – the "valley" in the gyroscope graph. Select the analysis tool and record the average values and the errors for a_x, a_y, and ω_z. Include the screenshot of the plot with correct highlights in the analysis mode.

(7) Compute the magnitude of the total centripetal acceleration, a_c.

By now, you should have three different values for a_c and ω.

TASKS

(8) Create a neat table with columns a_c, ω, and ω^2. Don't forget the units.

(9) Plot a_c vs. ω and a_c vs. ω^2.

3.7 Analysis: Plots

The centripetal acceleration is given by the formula

$$a_c = \frac{v^2}{r} \tag{3.4}$$

and the tangential velocity is given by

$$v = \omega r \tag{3.5}$$

Combining these two, we get

$$a_c = \omega^2 r \tag{3.6}$$

Clearly, if you plot a_c vs. ω, you obtain a parabola, and if you plot a_c vs. ω^2, you obtain a straight line with a certain slope.

TASKS

(10) Insert a power-law trendline (Ax^B) to the plot of a_c vs. ω. Obtain the values A and B.

(11) Insert a linear trendline to the plot of a_c vs. ω^2. Obtain the slope hence the radius of rotation. Compare it to the measured value.

The acceleration components and the angular velocity come with instrument errors. These can be used to estimate the uncertainty in the measured value of the radius.

TASKS

(12) Estimate the error in the radius by inserting error bars in the plot of a_c vs. ω^2.

3.8 Discussion

In the Conclusion and Discussion section of your lab report, include the following among other things (see Figure 3):

TASKS

(13) Look at whether any of a_x and a_y are positive or negative. Using the coordinate system given on the device and the position of the points `A` and `G`, explain if this makes sense.

(14) Compare the values obtained and measured for the radius.

3.9 Submitting lab report

In order not to experience any difficulties with the submission, please name your file as

`p121_exp3_lastname_firstname.pdf`

or

`p113_exp3_lastname_firstname.pdf`

minding the underscores. Then, click here to submit your paper [1].

[1] If the link does not work, please send your pdf to `kagannsimsek@gmail.com` by typing the subject of the email exactly as Mechanics Experiment 3. You may leave the body of the email empty.

Experiment 4

Momentum and energy

4.1 Objectives

In this experiment, we are going to

- learn how energy is transformed from one form to another,
- learn what factors can take energy away from the system,
- learn what a phase space plot is and apply it to the study of energy conservation,
- study the relation between impulse and change in momentum, and
- derive this relation from the Newton 2^{nd} Law.

4.2 Useful links

Here are some links that contain useful information about the theory behind this experiment and instructions on how to take and analyze data.

- Bullet and the block
- Batman – Joker
- Energy conservation
- Finding the mass of the device
- Momentum and impulse

4.3 Physics overview

4.3.1 Conservation of energy

Energy is conserved in an isolated system under no external force. However, this does not mean that its form will remain the same. In this experiment, we will see the transfer between kinetic energy and spring potential energy.

4.3.2 Momentum and impulse

Classically speaking, an object has a non-zero momentum if it has mass and velocity. Thus, if there is motion, there is momentum. Generally speaking, momentum is conserved if there are no external forces acting on the body/system.

4.4 Phase space plots

Mostly, we are accustomed to working with the plots of position versus time or velocity versus time. If you plot position versus velocity (or, equivalently, momentum), then you obtain a phase space plot. Each pair of the values (x, v) or (x, p) is a *state* of the system, and the combination of such possible states make up the phase space. Most often, we stick to the momentum instead of velocity. However, the iOLab device can plot velocity instead of momentum.

When we attach a mass to a spring and introduce some initial disturbance to the system, both position and momentum oscillate in time:

$$x = A\cos\omega t \tag{4.1}$$

$$p = B\sin\omega t \tag{4.2}$$

When plotted against each other, the resulting phase space will be in the shape of a circle. The radius of the circle – determined by the energy – remains constant if there is no external force in the system, namely if energy is conserved:

$$E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 = \frac{B^2}{2m}\sin^2\omega t + \frac{1}{2}m\omega^2A^2\cos\omega t \tag{4.3}$$

and under suitable conditions, A and B are beautifully related as

$$B = -m\omega A \tag{4.4}$$

(you can derive this by using dimensional analysis, but you may miss the negative sign, and that's totally okay) so you see the coefficients are the same in the energy expression, yielding a circle instead of an ellipse.

There are two types of energy that we will be looking at: kinetic energy and spring potential energy. We will ignore the gravitational effects.

4.5 Conservation of energy

Launch the iOLab application and pair the remote. Attach the eyebolt to the force probe, and attach the long spring to it. Tilt a table so that the table top is in a vertical position. In the iOLab application, select the sensors `Wheel-Position` and `Wheel-Velocity`. Make sure the wheels are in contact with the table. Start recording data. Allow the device to oscillate for a while. Stop recording data.

TASKS

(1) Parametrically plot velocity vs. position. Include a screenshot of your own graph in your report. You may notice that the figure is not a circle but an inward spiral. What conclusion can you draw about the energy conservation?

4.6 Finding the "known" value of the mass

The mass of the iOLab device will be measured in later again, hence you should learn how to perform this section quickly. Please, in the next experiment, do not use the data from this experiment.

Reset any previous runs. Insert the eyebolt into the force probe. Put the remote on the table so that the y axis is downwards and start recording data. Wait two seconds. Now grab the remote by the eyebolt and lift it. Hold steady for two seconds again. Put it back. Stop recording data.

TASKS

(2) Take a screenshot of the plots. Include them in your report.

(3) By using the analysis tool (to the left of the magnifier), select the "valley" in the force-time graph and record the average force and its uncertainty. Include the screenshot of your plot with correct highlights.

(4) By using the analysis tool, select the acceleration in the force-free region and record the average acceleration and its uncertainty. Include the screenshot of your plot with correct highlights.

(5) Compute the mass of the remote, and the uncertainty in it.

4.7 Momentum and impulse

Detach the long spring and the eyebolt off the force probe, and attach the short spring to the same spot. Put the device on its wheels (so that the z axis points downwards). Take a solid object – say, the box of the device. Hold it about 20 cm from the device. The spring should point towards the box (so that the y axis points away from the box). Give it a nice shove so that it bounces off the box.

We will use the sensors `Force` and `Wheel-Velocity`. Start recording data, and let the device bounce off the box as described above. Stop recording data.

TASKS

(6) Obtain the force versus time and velocity versus time plots. Include them in your report.

(7) Zoom into the peak in the force graph and by using the analysis tool, measure the area under the force-time plot. Include a screenshot of all you graphs with correct highlights.

(8) By using the analysis tool, measure the average of the initial and final velocities, together with the uncertainties. Include a screenshot of all your graphs with correct highlights.

(9) By using the mass of the device, compute the initial and final momenta.

(10) Compute the change in the momentum.

4.8 Problems

Answer the following now:

TASKS

(11) Does the radius of the phase space plot remain constant? Why or why not?

(12) What does this mean about the total amount of energy in the system? Is energy conserved?

(13) How did the area under the force versus time plot compare to the change in the velocity multiplied by the mass?

(14) Using the Newton 2$^{\mathrm{nd}}$ Law, the relation between acceleration and velocity, and the fact that $J = \int F \, \mathrm{d}t$, derive the relation between impulse and change in

momentum. Use this to explain your results from the second part (momentum-impulse) of the lab.

4.9 Error analysis

In the second part of this lab, you can obtain the error in the average velocity. Use it to compute the uncertainty in the change of velocity:

$$\Delta(\Delta v) = \Delta v_i + \Delta v_f \tag{4.5}$$

Meantime, the mass also has some uncertainty, Δm, deriving from the error in the gravitational force and the acceleration, as we did earlier. Then, you can compute the uncertainty in the change in momentum.

TASKS

(15) Compute the uncertainty in the change in momentum, and check to see if the area under the force versus time plot fits within this uncertainty limit.

4.10 Submitting lab report

In order not to experience any difficulties with the submission, please name your file as

`p121_exp4_lastname_firstname.pdf`

or

`p113_exp4_lastname_firstname.pdf`

minding the underscores. Then, click here to submit your paper [1].

[1] If the link does not work, please send your pdf to `kagannsimsek@gmail.com` by typing the subject of the email exactly as Mechanics Experiment 4. You may leave the body of the email empty.

Experiment 5

Simple pendulum

5.1 Objectives

In this experiment, we are going to

- measure the period of a simple pendulum,
- experimentally determine the relation between the period, T, and the length, L, of a pendulum, and
- compare experimental results with theoretical values of the period of the pendulum.

5.2 Physics overview

A pendulum consists of a weight suspended from a pivot in a manner by which it can swing freely. In a simple pendulum, this pivot point is fixed and frictionless and the mass can be treated as a point object.

In the case of a simple pendulum, the only forces acting on the mass are the force of gravity and tension. This will result in a free-body diagram like the one in Figure 5.1. For simplicity, the force of gravity will be broken up into its components such as in Figure 5.2.

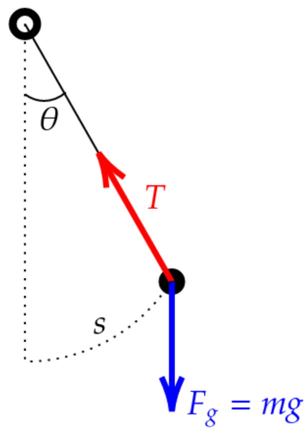


Figure 5.1: Forces acting on the point mass in a simple pendulum.

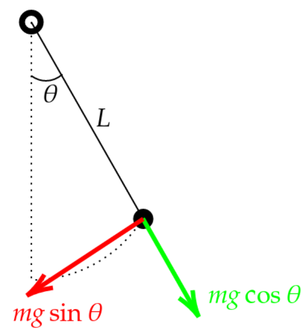


Figure 5.2: Components of the gravitational force acting on the point mass.

In this case, the component of the gravitational force, $mg\sin\theta$, is the one restoring the mass to its equilibrium position (this component of the gravitational force will always point toward the equilibrium position).

Using rotational kinematics, we can relate the torque to the angular acceleration:

$$\tau = I\alpha = I\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -mg\sin\theta\ L \tag{5.1}$$

$$mL^2\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -mg\sin\theta\ L \tag{5.2}$$

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{L}\sin\theta \tag{5.3}$$

Clearly, this is not simple harmonic motion. However, for small angles (and in radians), we can approximate $\sin\theta \approx \theta$ to yield simple harmonic motion:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{L}\theta \tag{5.4}$$

$$\omega^2 = \frac{g}{L} \tag{5.5}$$

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{5.6}$$

The period of this motion, denoted by T, is the time it takes for the mass to complete one full cycle, or to swing back and forth. We can use the known values of g and π to compute the theoretical values of T and compare these values to the values we measured.

5.3 Procedure

Hopefully, you are able to find a non-stretching string in your house. At worst, you can use a shoelace.

Find a piece of string, and tie it to the hanger at the top of the iOLab device (the exact opposite side where the force probe is). Securely tighten your fingers around the string at a fixed position, so that your fingers behave as the pivot point. Measure the distance, L, between this point and the point `G` on the device (Why?). Observe if you can keep your hand stable during the swing motion of the device (so that the pivot point does not move at all).

When you have practiced it a bit, launch the iOLab application, pair your device, and select the sensors `Acceleration` and `Gyroscope`. Deselect `Az` in the acceleration plot, and ω`x` and ω`y` in the gyroscope graph.

TASKS

(1) Measure the length of the pendulum, L, as describe above. Estimate the error, ΔL.

(2) Put your pivot hand in the air, start recording, and give the device a nice swing but do your best to ensure that the device swings in the xy plane only! Let the device swing some time. Then it will be a small oscillation towards the end. Notice that the x and y components of the acceleration oscillate by with a small amplitude in time. Stop recording data. Include a screenshot of the acceleration-time and angular velocity-time plots.

(3) By using the analysis tool, select at least three periods of the wave in the "small oscillation part" and record both the period, $T_{\text{exp}} = \Delta t$ (do not forget to divide Δt by the number of period you select), and the average and error of the x and y components of the acceleration. Include a screenshot of your plots with correct highlights.

Now, repeat it for a longer pendulum length.

TASKS

(4) Measure the length of the pendulum, L, as described above. Estimate the error, ΔL.

(5) Put your pivot hand in the air, start recording, and give the device a nice swing but do your best to ensure that the device swings in the xy plane only! Let the device swing some time. Then it will be a small oscillation towards the end. Notice that the x and y components of the acceleration oscillate by with a small amplitude in time. Stop recording data. Include a screenshot of the acceleration-time and angular velocity-time plots.

(6) By using the analysis tool, select at least three periods of the wave in the "small oscillation part" and record both the period, $T_{\rm exp} = \Delta t$ (do not forget to divide Δt by the number of period you select), and the average and error of the x and y components of the acceleration. Include a screenshot of your plots with correct highlights.

Now, repeat it for an even longer pendulum length.

TASKS

(7) Measure the length of the pendulum, L, as described above. Estimate the error, ΔL.

(8) Put your pivot hand in the air, start recording, and give the device a nice swing but do your best to ensure that the device swings in the xy plane only! Let the device swing some time. Then it will be a small oscillation towards the end. Notice that the x and y components of the acceleration oscillate by with a small amplitude in time. Stop recording data. Include a screenshot of the acceleration-time and angular velocity-time plots.

(9) By using the analysis tool, select at least three periods of the wave in the "small oscillation part" and record both the period, $T_{\rm exp} = \Delta t$ (do not forget to divide Δt by the number of period you select), and the average and error of the x and y components of the acceleration. Include a screenshot of your plots with correct highlights.

By now, you should have

- three different $L \pm \Delta L$ values,
- three different $a_x \pm \Delta a_x$ values,
- three different $a_y \pm \Delta a_y$ values, and

- three different T_{exp} values.

TASKS

(10) Prepare a neat table as follows:

$L \pm \Delta L$ (m)	$a_x \pm \Delta a_x$ (m/s^2)	$a_y \pm \Delta a_y$ (m/s^2)	$g \pm \Delta g$ (m/s^2)	$\frac{L}{g} \pm \Delta\left(\frac{L}{g}\right)$ (s^2)	T_{theo} (s)	T_{exp} (s)

where $g = \sqrt{a_x^2 + a_y^2}$, $\Delta g = (a_x \Delta a_x + a_y \Delta a_y)/g$, $\Delta(L/g) = (L/g)(\Delta L/L + \Delta g/g)$, $T_{\text{theo}} = 2\pi\sqrt{L/g}$, and T_{exp} is the experimental value of the period. Don't forget the units.

(11) Plot T_{exp} vs. L/g. Insert the error bars in your plot.

(12) Insert a power-law trendline of the form Ax^B. Obtain A and B.

(13) Compare A and B to the expected values.

As you may have already noticed, g here is the gravitational acceleration. It should be very close to the accepted value of $9.8\ \text{m/s}^2$.

5.4 Error analysis

Estimate the error in the formula you obtained for the relation between T_{exp} and L/g.

TASKS

(14) By using the formula you obtained for the relation between T_{exp} and L/g, compute the period at the length values of $L = 25\ \text{cm},\ 50\ \text{cm},\ \text{and}\ 75\ \text{cm}.$

(15) Compute the theoretical value of the period for the three length values in the previous step.

(16) Compute the relative percent error between the result obtained with your formula and the theoretical value with respect to the theoretical one for each length value, and obtain the average of the relative percent errors.

5.5 Discussion

In the Conclusion and Discussion section of your lab report, include the following among other things (see Figure 3):

TASKS

(17) Does the experiment agree with the theoretical predictions? Compute the error in the variables A and B.

(18) Can you suggest another (perhaps better) way of proving the relation among the period of the pendulum, its length, and the gravitational acceleration?

5.6 Submitting lab report

In order not to experience any difficulties with the submission, please name your file as

`p121_exp5_lastname_firstname.pdf`

or

`p113_exp5_lastname_firstname.pdf`

minding the underscores. Then, click here to submit your paper [1].

[1] If the link does not work, please send your pdf to `kagannsimsek@gmail.com` by typing the subject of the email exactly as Mechanics Experiment 5. You may leave the body of the email empty.

Formulae for error analysis

Each measurement comes with an error whose source may be the instrument itself or totally unknown. Technically speaking, the measured value is called the average and the error is referred to as the uncertainty in (or the standard deviation of) that measurement. Letting x denote the measured quantity, we write it as

$$x = x_{\mathrm{av}} \pm \Delta x \tag{5.7}$$

It is important to note that the plus/minus sign is nothing more than a symbol since we make the fundamental assumption that the standard deviation is always a positive quantity. This will allow us to play with it freely. This means we can treat the minus/plus sign to be the same thing as the original uncertainty, and as usual. However, in all the cases, we will favor the argument that whenever we perform an operation on quantities with errors in them, the error in the next step should never be smaller due to error propagation.

Now take two quantities that contain errors, say $x = x_{\mathrm{av}} \pm \Delta x$ and $y = y_{\mathrm{av}} \pm \Delta y$. Let's us derive the uncertainty in the most basic calculations. Let's start with the arithmetics.

Let z defined by the linear sum of the two quantities x and y. Then,

$$\begin{aligned} z &= z_{\mathrm{av}} \pm \Delta z \\ &= x + y \\ &= x_{\mathrm{av}} \pm \Delta x + y_{\mathrm{av}} \pm \Delta y \\ &= (x_{\mathrm{av}} + y_{\mathrm{av}}) \pm (\Delta x + \Delta y) \end{aligned} \tag{5.8}$$

and hence, using a more suggestive notation,

$$\boxed{\begin{aligned} (x+y)_{\mathrm{av}} &= x_{\mathrm{av}} + y_{\mathrm{av}} \qquad (5.9) \\ \Delta(x+y) &= \Delta x + \Delta y \qquad (5.10) \end{aligned}}$$

Next, let z defined by the difference in the two quantities x and y. Then,

$$\begin{aligned} z &= z_{\mathrm{av}} \pm \Delta z \\ &= x - y \end{aligned}$$

$$= x_{\text{av}} \pm \Delta x - y_{\text{av}} \mp \Delta y$$
$$= (x_{\text{av}} - y_{\text{av}}) \pm (\Delta x + \Delta y) \tag{5.11}$$

where in the last step, we have used the property that the minus/plus sign is completely equivalent to the plus/minus and the fact that the error should always accumulate or increase. Hence,

$$\boxed{(x-y)_{\text{av}} = x_{\text{av}} - y_{\text{av}}} \tag{5.12}$$
$$\boxed{\Delta(x+y) = \Delta x + \Delta y} \tag{5.13}$$

Next, let z defined by the multiplication of the two quantities x and y. Then,

$$\begin{aligned} z &= z_{\text{av}} \pm \Delta z \\ &= xy \\ &= (x_{\text{av}} \pm \Delta x)(y_{\text{av}} \pm \Delta y) \\ &= x_{\text{av}}y_{\text{av}} \pm x_{\text{av}}\Delta y \pm y_{\text{av}}\Delta x + \Delta x \Delta y \\ &= x_{\text{av}}y_{\text{av}} \pm x_{\text{av}}y_{\text{av}} \left(\frac{\Delta x}{x_{\text{av}}} + \frac{\Delta y}{y_{\text{av}}} + \frac{\Delta x}{x_{\text{av}}}\frac{\Delta y}{y_{\text{av}}} \right) \\ &= x_{\text{av}}y_{\text{av}} \pm x_{\text{av}}y_{\text{av}} \left(\frac{\Delta x}{x_{\text{av}}} + \frac{\Delta y}{y_{\text{av}}} \right) \end{aligned} \tag{5.14}$$

where we have ignored the term proportional to the product $\Delta x \Delta y$. We can justify this if we take into account the fact that uncertainties should be much smaller than averages. For instance, an acceptable percentage of error is around 5% in engineering and much less than 1% in optics. Thus, the first and second terms in the parentheses in Equation (5.14) are around, say, 1 to 5% but the last term is around 0.01 to 0.25% generally in the aforementioned fields. This is why the last term in the parentheses can be safely ignored.

There is one final word of caution here: Even if we assume that the standard deviation is always positive, the values x_{av} and y_{av} may not. It is okay if both x_{av} and y_{av} are positive or negative; it is problematic only when they have the opposite signs, which would decrease the error. Thus, we need to introduce absolute value just to be sure. H ence,

$$\boxed{(xy)_{\text{av}} = x_{\text{av}}y_{\text{av}}} \tag{5.15}$$
$$\boxed{\Delta(xy) = |x_{\text{av}}y_{\text{av}}| \left(\left|\frac{\Delta x}{x_{\text{av}}}\right| + \left|\frac{\Delta y}{y_{\text{av}}}\right| \right)} \tag{5.16}$$

Next, let z be defined as the ratio of the two quantities x and y. Then,

$$z = z_{\text{av}} \pm \Delta z = \frac{x_{\text{av}} \pm \Delta x}{y_{\text{av}} \pm \Delta y} \tag{5.17}$$

It would be much easier if the term in the denominator had been in the numerator as a factor multiplying the first term. There is a nice trick for that, and it employs the binomial approximation[2]. For a real number a such that $|a| \ll 1$, we have

$$(1+a)^{-1} \approx 1 - a \tag{5.18}$$

Then,

$$\frac{1}{y_{\text{av}} \pm \Delta_y} = \frac{1}{y_{\text{av}}\left(1 \pm \frac{\Delta y}{y_{\text{av}}}\right)} = \frac{1}{y_{\text{av}}} \frac{1}{1 \pm \frac{\Delta y}{y_{\text{av}}}} \tag{5.19}$$

The second factor here meets the properties as a above: $\Delta y/y$ is a real number with magnitude much less than 1. So, we can use the aforementioned trick to write

$$\frac{1}{y_{\text{av}} \pm \Delta y} = \frac{1}{y_{\text{av}}}\left(1 \mp \frac{\Delta y}{y_{\text{av}}}\right) = \frac{1}{y_{\text{av}}}\left(1 \pm \frac{\Delta y}{y_{\text{av}}}\right) \tag{5.20}$$

where in the final step we have used the fact that the minus/plus sign is completely equivalent to the plus/minus. Inserting this into Equation (5.17), we obtain

$$\begin{aligned}
z_{\text{av}} \pm \Delta z &= (x_{\text{av}} \pm \Delta x)\frac{1}{y_{\text{av}}}\left(1 \pm \frac{\Delta y}{y_{\text{av}}}\right) \\
&= x_{\text{av}}\left(1 \pm \frac{\Delta x}{x_{\text{av}}}\right)\frac{1}{y_{\text{av}}}\left(1 \pm \frac{\Delta y}{y_{\text{av}}}\right) \\
&= \frac{x_{\text{av}}}{y_{\text{av}}}\left(1 \pm \frac{\Delta x}{x_{\text{av}}} \pm \frac{\Delta y}{y_{\text{av}}} + \frac{\Delta x}{x_{\text{av}}}\frac{\Delta y}{y_{\text{av}}}\right) \\
&= \frac{x_{\text{av}}}{y_{\text{av}}}\left(1 \pm \frac{\Delta x}{x_{\text{av}}} \pm \frac{\Delta y}{y_{\text{av}}}\right) \\
&= \frac{x_{\text{av}}}{y_{\text{av}}} \pm \frac{x_{\text{av}}}{y_{\text{av}}}\left(\frac{\Delta x}{x_{\text{av}}} + \frac{\Delta y}{y_{\text{av}}}\right)
\end{aligned} \tag{5.21}$$

where we have ignored the last term in the parentheses in the penultimate step and by now, you should know why. Finally, inserting the useful absolute values, we get

$$\left(\frac{x}{y}\right)_{\text{av}} = \frac{x_{\text{av}}}{y_{\text{av}}} \tag{5.22}$$

$$\Delta\left(\frac{x}{y}\right) = \left|\frac{x_{\text{av}}}{y_{\text{av}}}\right|\left(\left|\frac{\Delta x}{x_{\text{av}}}\right| + \left|\frac{\Delta y}{y_{\text{av}}}\right|\right) \tag{5.23}$$

Next, let's do the square[3] Suppose z is defined by taking the square of x. By using Equations (5.15) and (5.16) (and I leave the tiny step of simplification to you), we get

$$(x^2)_{\text{av}} = x_{\text{av}}^2 \tag{5.24}$$

$$\Delta(x^2) = |2x_{\text{av}}\Delta x| \tag{5.25}$$

[2] If you know what a Taylor series is, then you can perform the explicit derivation by keeping the first-order terms only. Or else, stick to this "trick."

[3] I don't think you may ever encounter or need this in your labs, but for the third power, once you know z^2, you can obtain it by multiplying z by z^2, so the procedure is quite straightforward.

Finally, let's do the square root and complete this chapter. Suppose z is given by the square root of x:

$$z = z_{\rm av} \pm \Delta z = \sqrt{x_{\rm av} \pm \Delta x} \tag{5.26}$$

We are going to use the binomial approximation again. Let me quote the most general trick:

$$(1+a)^n \approx 1 + na \tag{5.27}$$

for any real number a such that $|a| \ll 1$. Then, it is obvious that

$$\begin{aligned} \sqrt{x_{\rm av} \pm \Delta x} &= (x_{\rm av} \pm \Delta x)^{1/2} \\ &= x_{\rm av}^{1/2} \left(1 \pm \frac{\Delta x}{x_{\rm av}}\right)^{1/2} \\ &\approx \sqrt{x_{\rm av}} \left(1 \pm \frac{\Delta x}{2x_{\rm av}}\right) \\ &= \sqrt{x_{\rm av}} \pm \frac{\Delta x}{2\sqrt{x_{\rm av}}} \end{aligned} \tag{5.28}$$

By taking the absolute values, we finally get

$$(\sqrt{x})_{\rm av} = \sqrt{x_{\rm av}} \tag{5.29}$$

$$\Delta(\sqrt{x}) = \frac{|\Delta x|}{2\sqrt{|x_{\rm av}|}} \tag{5.30}$$