2022 Summer Phys 135-2: Questions from students for the midterm

(Dated: July 19, 2022)

(Last updated: September 14, 2022)

Abstract

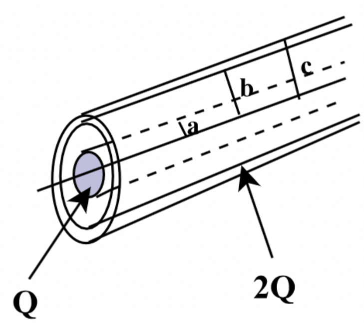
In this paper, we discuss the common problems brought upon by the students for the midterm of the course Physics 135-2 in the summer quarter of the academic year 2021-2022.

CONTENTS

I. ELECTRIC FIELD CREATED BY A SOLID, CONDUCTING CYLINDER SURROUNDED BY A CONCENTRIC, NONCONDUCTING CYLINDRICAL SHELL

The problem is given as follows:

2. A long, solid, conducting cylinder of radius $a = 1.0$ cm and length $L = 5.0$ m carries excess positive charge Q = + 3.0 nC. Centered on the axis of this solid cylinder is a nonconducting cylindrical shell, also of length $L = 5.0$ m, that has excess charge 2Q = + 6.0 nC spread uniformly from inner radius b = 2.0 cm to outer radius $c = 2.5$ cm.



a. Calculate the electric field strength at a radial distance r from the central axis, where $r = 0.5$ cm.

b. Calculate the electric field strength at a radial distance r from the central axis, where $r = 1.5$ cm.

c. Calculate the electric field strength at a radial distance r from the central axis, where $r = 2.25$ cm.

FIG. 1.

Nobody asked a question on the details of the calculations regarding the usage of the Gauss law, so it seems safe to assume that everybody is okay with that. However, there appears to be a confusion about the last part.

As many of the students indicated, the confusion seems to originate from using a linear charge density for the conductor and a volumetric charge density for the insulating shell and the question is *why?*. The short answer is *why not!*. It doesn't matter how you compute the charge enclosed in a given region as long as you come with the correct result. A more detailed answer would include the fact that in the insulating shell, the charge $2Q$ will be distributed throughout the shell in a uniform manner, covering the entire thickness of the shell. Thus, in a way, we *have to* use the volumetric charge density. As for the conducting inner core, we know that the charge Q will be distributed only at the surface defined by $r = a$, again uniformly. There is no volumetric density to consider here because it is a surface charge density at best. We may define $\sigma = \frac{Q}{\pi a^2 \times L}$ and while computing the charge enclosed for this part, we will have

$$q_{\text{enc}} = \text{area} \times \text{surface density} = (\pi r^2 \times \ell) \times \frac{Q}{\pi r^2 \times L} \tag{1}$$

which gives the same result as with the linear charge density. There is nothing profound here.

We suggest that if you have more specific questions about this problem, please contact the author.

II. ELECTRON BETWEEN TWO CHARGED PARTICLES

The problem is given below:

1. An electron is placed midway between two charges q_1 and q_2 that are separated by a distance $d = 3.0$ μm, as in the diagram below. (Note: 1 μm = 10^{-6} m.) Charge #1 is positive and charge #2 is negative. The electrostatic potential at the midpoint due to both #1 and #2 is 0.959 mV. (Note: 1 mV = 10^{-3} V) If the electron has instantaneous acceleration of 1.01×10^{15} m/s^2 at the midpoint, what are the numerical values of q_1 and q_2 ?

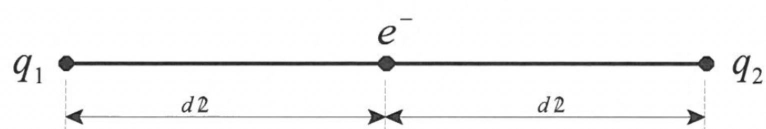


FIG. 2.

Let's summarize what's going on in this problem and leave the algebra to the reader. We are asked to find the two unknowns, q_1 and q_2, by using the info regarding the potential at the point where the electron is located and the instantaneous acceleration of the electron. We have two equations, so it will be an easy task.

Advice: Avoid using numerical values in the analysis and, if the problem does not tell you to evaluate a quantity at some intermediate point, insert numbers at the very end. This will help us catch your mistakes, if any, while grading your papers. It is also beneficial to keep track of variables and perform sanity checks in the middle of calculations to see if you forget dividing or multiplying by something by means of, for example, dimensional analysis. It's good practice and take this advice seriously.

Let's start with the potential. We may put the origin on the charge q_1. The electron is supposed to accelerate, so we want to avoid such an accelerating reference frame. Suppose we align the positive x axis toward q_2 from there. Then, the x-component of the position of the particles are

$$x_1 = 0, \quad x_e = \frac{d}{2}, \quad x_2 = d \tag{2}$$

The potential at the midpoint between q_1 and q_2 due to said charges is

$$V_e = \frac{kq_1}{\frac{d}{2}} + \frac{kq_2}{\frac{d}{2}} = +\frac{2k|q_1|}{d} - \frac{2k|q_2|}{d} \tag{3}$$

where we emphasized the sign of the charges. This gives a relation in terms of q_1 and q_2:

$$|q_1| - |q_2| = \frac{V_e d}{2k} \tag{4}$$

We are also given the acceleration of the electron at this position. Newton says $\boldsymbol{F} = m_e \boldsymbol{a}_e$ and we know that $\boldsymbol{F} = -e\boldsymbol{E}_e$, where

$$\boldsymbol{E} = kq_1 \frac{\frac{d}{2} - 0}{\left|\frac{d}{2} - 0\right|^3} + kq_2 \frac{\frac{d}{2} - d}{\left|\frac{d}{2} - d\right|^3} = \frac{4k|q_1|}{d^2} + \frac{4k|q_2|}{d^2} \tag{5}$$

Then, we get

$$|\boldsymbol{a}_e| = \frac{|-e\boldsymbol{E}|}{m_e} = \frac{e}{m_e}\frac{4k(|q_1| + |q_2|)}{d^2} \tag{6}$$

which gives

$$|q_1| + |q_2| = \frac{m_e|\boldsymbol{a}_e|d^2}{4ke} \tag{7}$$

We have Eqs. (4) and (7) in $|q_1|$ and $|q_2|$, which are now really straightforward to solve. It's essential that you get the idea and apply it correctly. Once you know what you are doing, any numerical mistakes should be tolerable.

III. TWO CAPACITORS: ARE THEY IN PARALLEL OR SERIES?

The three infamous problems that have caused mass confusion are given below:

Problem C - 2

A potential difference of 300 V is applied to a series connection of capacitance $C_1 = 2.0\ \mu F$ and capacitance $C_2 = 8.0\ \mu F$. (a) What is the charge and the potential difference for each capacitor? (b) The charged capacitors are disconnected from each other and from the battery. While still charged, they are reconnected, positive plate to positive plate and negative plate to negative plate. The battery is not used. After the charge redistributes, what is the charge and the potential difference for each capacitor? (c) Suppose the charged capacitors in (a) were reconnected with plates of *opposite* sign together. What then would be the steady-state charge and potential difference for each?

FIG. 3.

Problem C - 7

The figure at right shows a 12.0 V battery and three uncharged capacitors: $C_1 = 4.00\ \mu F$, $C_2 = 6.00\ \mu F$, and $C_3 = 3.00\ \mu F$. The switch is thrown to the left side until capacitor 1 is fully charged. Then the switch is thrown to the right. What is the final charge on (a) capacitor 1, (b) capacitor 2, and (c) capacitor 3?

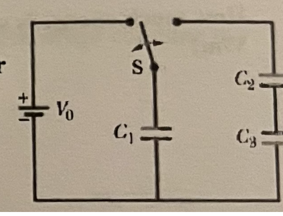


FIG. 4.

Problem C - 8

In the figure, capacitors $C_1 = 1.0\ \mu F$ and $C_2 = 3.0\ \mu F$ are each charged to a potential difference of $V = 100$ V but with opposite polarity. If we close switches S_1 and S_2, find the charge Q_1 on C_1 and the charge Q_2 on C_2 after a long time has passed?

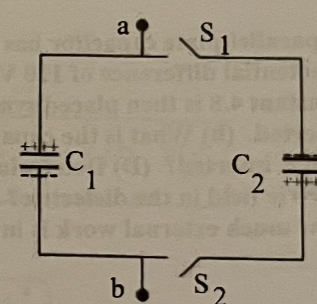


FIG. 5.

A confusion emerges on our perception of the connection type of these capacitors. *Are they connected in parallel or in series?* and *how do we decide on the connection?* are the two most common questions brought upon by the students. The answer is a bit tricky but when you think *really hard* about it, it will sound highly intuitive.

Think about the reason why you may not be able to easily identify or decide on the type of connection that takes places between two conductors, for instance, for part (b) of the first problem, i.e. Problem C-2. What causes this confusion? Would include a resistance or a battery help you remove the confusion?

The answer to these questions resides in the fact that when we construct a circuit using two elements of the same type (resistance, capacitor, inductor, power supply) and using only these, parallel and series connections lose their meaning. They become ambiguous — well, we think they become ambiguous because at first, we don't understand what's going on. The reason that we can't really talk about parallel and series connections in the familiar sense is that there is no reference point or circuit element that will render our assumption true or false. Consider part (b) of the second problem or the third problem. Would it help if we throw in a battery or a resistance to the circuit? Yes! That's the answer. In the presence of a power supply, we tend to imagine the direction of a current. If we connect a resistor (say a light bulb), on the other hand, we'll know that some capacitors will produce a current to feed the resistor to light it up. These all sound natural. But what's going on with just two capacitors?

Suppose you connect two capacitors together end to end using two wires:

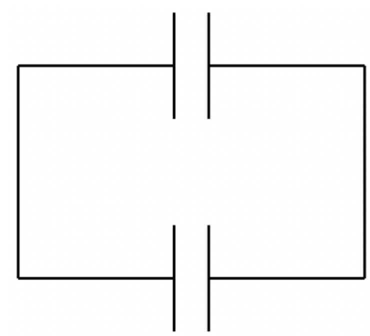


FIG. 6.

Notice that we didn't say anything about the polarity of the capacitors. Now, let's see what's going on here. Let's assume the upper one is C_1 and the lower one is C_2. Suppose $C_1 \neq C_2$. The left vertical line of each capacitor are connected by a wire on the left part of the diagram. So, we have two metal plates connected by a conducting wire. When we connect two charged, conducting bodies, charges will flow around so that the two objects have the same potential at their surfaces. This is what happens with these two connected plates. Let's call this potential V_L. A similar consideration is valid for the plates connected by the right branch. Call this potential V_R. Now, each conductor has a potential difference $V_L - V_R$. Thus, in the equilibrium, we can easily associate this configuration with two parallel-connected capacitors. Easy, isn't it?

You may ask *what happens if the voltage values are the same to begin with?* and that's a perfectly valid question. Then, we'll look at the charges. Suppose the left plate of the upper (lower) capacitor carries an initial charge Q_1 (Q_2). Then, the total net charge on the wire that connects the two plates is $Q_{\text{net}} = Q_1 + Q_2$. The wire on the right branch will carry the same charge with a minus sign. Note that we still don't talk about the polarities. Actually, this info is hidden in the charges. Q_1 and Q_2 may both have the same sign or they may have the opposite signs. That's totally okay. After determining the net charge, we analyze the system so that the final potential sees an equivalent capacitor, which is made up of two parallel-connected capacitors, having a charge Q_{net}. The rest can be followed from the Red Book.

Let's solve these three problems together now without digging much into algebra. In the first problem, in part (a), we connect one capacitor after the other but now, instead of connecting the capacitors with a second wire, we put a battery in between. Notice how the battery breaks the ambiguity of the connection so that we are able to conclude that this connection is a series connection. The two capacitors will get charged up. When they are fully charged, they will carry the same amount of charge but the external voltage will be shared by the two capacitors. Next, in the second part, part (b), we connect them such that a positive end talks to a positive end and a negative end talks to a negative end. This is the configuration which we discussed just above but with $Q_1 = Q_2 = Q$. Since this is effectively a parallel connection, we add up the capacitance values to get C_{eq}. Then, by requiring the voltage values to be equal in the final state, we write down the equations as given in the manual. The key step is to realize that this is a parallel connection. The rest is just algebra. In part (c), it asks what happens if we connect the positive ends to the negative ones. Well, we have the same configuration pictured above but now with $Q_1 = -Q_2 = Q$ so that the net charge vanishes. If there is no charge, then there is no voltage. The charges on the plates will just cancel each other upon connection.

Let's do the third problem now because the second one is a bit involved — not because it is hard but we just have to deal with a third capacitor. Now, in this problem, we are given the same diagram that we drew above but with opposite polarities. In this case, the initial voltage values are the same. Note that this has nothing to do with the determination of the connection type of these two capacitors. That is, we don't say these are connected just because they have the same voltage to begin with. The same logic as in the first problem applies here. The capacitors have different Q_1 and Q_2 values, with even opposite signs, so we move on to the computation of the net charge, which is given by $Q_1 - Q_2$, depending on the branch you are looking at — the upper one or the lower one. It won't matter, though. As long as you work with positive values for the capacitance, you should be all okay. After determine the net charge on each branch, we move on to the computation of the equivalent capacitance, which is again the sum of the two capacitance values. Then we write down the potential and charge equations and the rest is algebra, as shown in the method. We believe that once you convince yourselves about the connection type, you won't have a problem with the rest of the calculations.

Now the second problem. At first, we charge up the first capacitor using some voltage value. Now it carries a certain amount of charge. Then, we turn the switch to the right to cancel out the battery and to direct the current to the two capacitors in the vertical branch. You (should) see that the two capacitors on the right are connected in *series* just because we have a reference circuit element. You may get tempted to assume a series connection between C_1 and C_2 and connect C_3 in parallel to them but that'd be wrong. C_1 is the charged one: it has a certain amount of charge, say $+Q$, on its top plate (because that plate was the one that was connected to the positive end of the power supply before we cut the battery out) and $-Q$ on the bottom plate. However, C_2 and C_3 are uncharged. Hence, the equivalent capacitor, say a single capacitor, C_{23}, to replace C_2 and C_3, will also have zero charge on its plates, so we say that C_2 and C_3 are compatible to each other. After combining these two capacitors in series, we'll know that they will carry the same amount of charge once C_1 releases and redistributes some of its charge. Again we have the same picture depicted above but with $Q_1 = Q$ (or whatever the charge the first capacitors holds) and $Q_{23} = 0$ initially. We'll then watch the charges redistribute themselves so that in the final state, C_1 and C_{23} have the same potential. Once you know the potential and the charge of the equivalent capacitor C_{23}, you know how to distribute these into its components from the first problem, Problem C-2.

The biggest source of confusion that the students have experience is that the two capacitors are connected in such a way that one's positive end meets the other one's negative end. This has lead many students to assume that the two capacitors must have been connected in series. In a more populated circuit containing other types of elements, that would be true because in such an environment, we get the sense of a flow of current

— we know where the current splits and meets again. But, as discussed in this section, with only two capacitors, we have to look into other details and shouldn't make blind assumptions.

If you still have questions about this issue of the connection type of two capacitors, patiently work out Problem C-1 in this framework again. It was a straightforward question, but now you know that a circuit with only two capacitors can be thought of as having a parallel connection. It may worth a while to go over this again.